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Diversity

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12.1 Introduction

Diversity is a commonly used technique in mobile radio systems to combat signal **fading.** The basic principle of diversity is as follows. If several replicas of the same information-carrying signal are received over multiple channels with comparable strengths, which exhibit independent fading, then there is a good likelihood that at least one or more of these received signals will not be in a fade at any given instant in time, thus making it possible to deliver adequate signal level to the receiver. Without diversity techniques, in noise limited conditions, the transmitter would have to deliver a much higher power level to protect the link during the short intervals when the channel is severely faded. In mobile radio, the power available on the reverse link is severely limited by the battery capacity of hand-held subscriber units. Diversity methods play a crucial role in reducing transmit power needs. Also, cellular communication networks are mostly interference limited and, once again, mitigation of channel fading through use of diversity can translate into reduced variability of carrier-to-interference ratio (C/I), which in turn means lower C/I margin and hence better reuse factors and higher system capacity.

The basic principles of diversity have been known since 1927 when the first experiments in space diversity were reported. There are many techniques for obtaining independently fading branches, and these can be subdivided into two main classes. The first are explicit techniques where explicit redundant signal transmission is used to exploit diversity channels. Use of dual polarized signal transmission and reception in many point-to-point radios is an example of explicit diversity. Clearly such redundant signal transmission involves a penalty in frequency spectrum or additional power. In the second class are implicit diversity techniques: the signal is transmitted only once, but the decorrelating effects in the propagation medium such as multipaths are exploited to receive signals over multiple diversity channels. A good example of implicit diversity is the **RAKE receiver** in code division multiple access (CDMA) systems, which uses independent fading of resolvable multipaths to achieve diversity gain. Figure 12.1 illustrates the principle of diversity where two independently fading signals are shown along with the selection diversity output signal which selects the stronger signal. The fades in the resulting signal have been substantially smoothed out while also yielding higher average power.

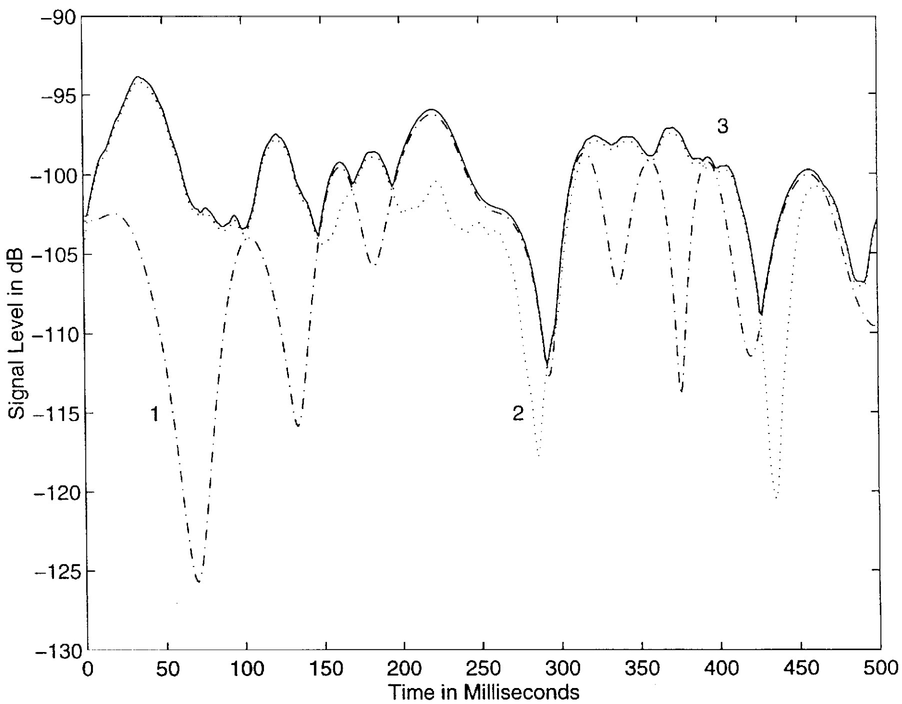

FIGURE 12.1: Example of diversity combining. Two independently fading signals 1 and 2. The signal 3 is the result of selecting the strongest signal.

If antennas are used in transmit, they can be exploited for diversity. If the transmit channel is known, the antennas can be driven with complex conjugate channel weighting to co-phase the signals at the receive antenna. If the forward channel is not known, we have several methods to convert space selective fading at the transmit antennas to other forms of diversity exploitable in the receiver.

Exploiting diversity needs careful design of the communication link. In explicit diversity, multiple copies of the same signal are transmitted in channels using either a frequency, time, or polarization dimension. At the receiver end we need arrangements to receive the different diversity branches (this is true for both explicit and implicit diversity). The different diversity branches are then combined to reduce signal **outage probability** or bit error rate.

In practice, the signals in the diversity branches may not show completely independent fading.

The envelope cross correlation ρ between these signals is a measure of their independence.

$$\rho = \frac{E\left[[r_1 - \bar{r}_1][r_2 - \bar{r}_2]\right]}{\sqrt{E\left|r_1 - \bar{r}_1\right|^2 E\left|r_2 - \bar{r}_2\right|^2}}$$

where r_1 and r_2 represent the instantaneous envelope levels of the normalized signals at the two receivers and $\bar{r_1}$ and $\bar{r_2}$ are their respective means. It has been shown that a cross correlation of 0.7 [3] between signal envelopes is sufficient to provide a reasonable degree of diversity gain. Depending on the type of diversity employed, these diversity channels must be sufficiently *separated* along the appropriate diversity dimension. For spatial diversity, the antennas should be separated by more than the *coherence distance* to ensure a cross correlation of less than 0.7. Likewise in frequency diversity, the frequency separation must be larger than the *coherence bandwidth*, and in time diversity the separation between channel reuse in time should be longer than the *coherence time*. These coherence factors in turn depend on the channel characteristics. The coherence distance, coherence bandwidth and coherence time vary inversely as the angle spread, delay spread, and Doppler spread, respectively.

If the receiver has a number of diversity branches, it has to combine these branches to maximize the signal level. Several techniques have been studied for diversity combining. We will describe three main techniques: selection combining, equal gain combining, and maximal ratio combining.

Finally, we should note that diversity is primarily used to combat fading and if the signal does not show significant fading in the first place, for example when there is a direct path component, diversity combining may not provide significant diversity gain. In the case of antenna diversity, array gain proportional to the number of antennas will still be available.

12.2 Diversity Schemes

There are several techniques for obtaining diversity branches, sometimes also known as diversity dimensions. The most important of these are discussed in the following sections.

12.2.1 Space Diversity

This has historically been the most common form of diversity in mobile radio base stations. It is easy to implement and does not require additional frequency spectrum resources. Space diversity is exploited on the reverse link at the base station receiver by spacing antennas apart so as to obtain sufficient decorrelation. The key for obtaining minimum uncorrelated fading of antenna outputs is adequate spacing of the antennas. The required spacing depends on the degree of multipath angle spread. For example if the multipath signals arrive from all directions in the azimuth, as is usually the case at the mobile, antenna spacing (coherence distance) of the order of 0.5λ to 0.8λ is quite adequate [5]. On the other hand if the multipath angle spread is small, as in the case of base stations, the coherence distance is much larger. Also empirical measurements show a strong coupling between antenna height and spatial correlation. Larger antenna heights imply larger coherence distances. Typically 10λ to 20λ separation is adequate to achieve $\rho = 0.7$ at base stations in suburban settings when the signals arrive from the broadside direction. The coherence distance can be 3 to 4 times larger for endfire arrivals. The endfire problem is averted in base stations with trisectored antennas as each sector needs to handle only signals arriving $\pm 60°$ off the broadside. The coherence distance depends strongly on the terrain. Large multipath angle spread means smaller coherence distance. Base stations normally use space diversity in the horizontal plane only. Separation in the vertical plane can also be used, and the necessary spacing depends upon vertical multipath angle spread. This can be small for distant mobiles making vertical plane diversity less attractive in most applications.

Space diversity is also exploitable at the transmitter. If the forward channel is known, it works much like receive space diversity. If it is not known, then space diversity can be transformed to another form of diversity exploitable at the receiver. (See Section 12.2.7 below).

If antennas are used at transmit and receive, the M transmit and N receive antennas both contribute to diversity. It can be shown that if simple weighting is used without additional bandwidth or time/memory processing, then maximum diversity gain is obtained if the transmitter and receiver use the left and right singular vectors of the $M \times N$ channel matrix, respectively. However, to approach the maximum $M \times N$ order diversity order will require the use of additional bandwidth or time/memory-based methods.

12.2.2 Polarization Diversity

In mobile radio environments, signals transmitted on orthogonal polarizations exhibit low fade correlation, and therefore, offer potential for diversity combining. Polarization diversity can be obtained either by explicit or implicit techniques. Note that with polarization only two diversity branches are available as against space diversity where several branches can be obtained using multiple antennas. In explicit polarization diversity, the signal is transmitted and received in two orthogonal polarizations. For a fixed total transmit power, the power in each branch will be 3 dB lower than if single polarization is used. In the implicit polarization technique, the signal is launched in a single polarization, but is received with cross-polarized antennas. The propagation medium couples some energy into the cross-polarization plane. The observed cross-polarization coupling factor lies between 8 to 12 dB in mobile radio [8, 1]. The cross-polarization envelope decorrelation has been found to be adequate. However, the large branch imbalance reduces the available diversity gain.

With hand-held phones, the handset can be held at random orientations during a call. This results in energy being launched with varying polarization angles ranging from vertical to horizontal. This further increases the advantage of cross-polarized antennas at the base station since the two antennas can be combined to match the received signal polarization. This makes polarization diversity even more attractive. Recent work [4] has shown that with variable launch polarization, a cross-polarized antenna can give comparable overall (matching plus diversity) performance to a vertically polarized space diversity antenna.

Finally, we should note that cross-polarized antennas can be deployed in a compact antenna assembly and do not need large physical separation needed in space diversity antennas. This is an important advantage in the **PCS** base stations where low profile antennas are needed.

12.2.3 Angle Diversity

In situations where the angle spread is very high, such as indoors or at the mobile unit in urban locations, signals collected from multiple nonoverlapping beams offer low fade correlation with balanced power in the diversity branches. Clearly, since directional beams imply use of antenna aperture, angle diversity is closely related to space diversity. Angle diversity has been utilized in indoor wireless LANs, where its use allows substantial increase in LAN throughputs [2].

12.2.4 Frequency Diversity

Another technique to obtain decorrelated diversity branches is to transmit the same signal over different frequencies. The frequency separation between carriers should be larger than the coherence bandwidth. The coherence bandwidth, of course, depends on the multipath delay spread of the channel. The larger the delay spread, the smaller the coherence bandwidth and the more closely we can space the frequency diversity channels. Clearly, frequency diversity is an explicit diversity technique and needs additional frequency spectrum.

A common form of frequency diversity is multicarrier (also known as multitone) modulation. This technique involves sending redundant data over a number of closely spaced carriers to benefit from frequency diversity, which is then exploited by applying **interleaving** and **channel coding/forward error correction** across the carriers. Another technique is to use **frequency hopping** wherein the interleaved and channel coded data stream is transmitted with widely separated frequencies from burst to burst. The wide frequency separation is chosen to guarantee independent fading from burst to burst.

12.2.5 Path Diversity

This implicit diversity is available if the signal bandwidth is much larger than the channel coherence bandwidth. The basis for this method is that when the multipath arrivals can be resolved in the receiver and since the paths fade independently, diversity gain can be obtained. In CDMA systems, the multipath arrivals must be separated by more than one *chip* period and the RAKE receiver provides the diversity [9]. In TDMA systems, the multipath arrivals must be separated by more than one *symbol* period and the MLSE receiver provides the diversity.

12.2.6 Time Diversity

In mobile communications channels, the mobile motion together with scattering in the vicinity of the mobile causes time selective fading of the signal with Rayleigh fading statistics for the signal envelope. Signal fade levels separated by the *coherence time* show low correlation and can be used as diversity branches if the same signal can be transmitted at multiple instants separated by the coherence time. The coherence time depends on the Doppler spread of the signal, which in turn is a function of the mobile speed and the carrier frequency.

Time diversity is usually exploited via interleaving, forward-error correction (FEC) coding, and **automatic request for repeat** (ARQ). These are sophisticated techniques to exploit channel coding and time diversity. One fundamental drawback with time diversity approaches is the delay needed to collect the repeated or interleaved transmissions. If the coherence time is large, as for example when the vehicle is slow moving, the required delay becomes too large to be acceptable for interactive voice conversation.

The statistical properties of fading signals depend on the field component used by the antenna, the vehicular speed, and the carrier frequency. For an idealized case of a mobile surrounded by scatterers in all directions, the autocorrelation function of the received signal $x(t)$ (note this is not the envelope $r(t)$) can be shown to be

$$E\left[x(t)x(t+\tau)\right] = J_0\left(2\pi\tau v/\lambda\right)$$

where J_0 is a Bessel function of the 0th order and v is the mobile velocity.

12.2.7 Transformed Diversity

In transformed diversity, the space diversity branches at the transmitter are transformed into other forms of diversity branches exploitable at the receiver. This is used when the forward channel is not known and shifts the responsibility of diversity combining to the receiver which has the necessary channel knowledge.

Space to Frequency

- *Antenna-delay.* Here the signal is transmitted from two or more antennas with delays of the order of a chip or symbol period in CDMA or TDMA, respectively. The different transmissions simulate resolved path arrivals that can be used as diversity branches by the RAKE or MLSE equalizer.
- *Multicarrier modulation.* The data stream after interleaving and coding is modulated as a multicarrier output using an inverse DFT. The carriers are then mapped to the different antennas. The space selective fading at the antennas is now transformed to frequency selective fading and diversity is obtained during decoding.

Space to Time

- *Antenna hopping/phase rolling.* In this method the data stream after coding and interleaving is switched randomly from antenna to antenna. The space selective fading at the transmitter is converted into a time selective fading at the receiver. This is a form of "active" fading.
- *Space-time coding.* The approach in space-time coding is to split the encoded data into multiple data streams each of which is modulated and simultaneously transmitted from different antennas. The received signal is a superposition of the multiple transmitted signals. Channel decoding can be used to recover the data sequence. Since the encoded data arrive over uncorrelated fade branches, diversity gain can be realized.

12.3 Diversity Combining Techniques

Several diversity combining methods are known. We describe three main techniques: selection, maximal ratio, and equal gain. They can be used with each of the diversity schemes discussed above.

12.3.1 Selection Combining

This is the simplest and perhaps the most frequently used form of diversity combining. In this technique, one of the two diversity branches with the highest carrier-to-noise ratio (C/N) is connected to the output. See Fig. 12.2(a).

The performance improvement due to selection diversity can be seen as follows. Let the signal in each branch exhibit Rayleigh fading with mean power σ^2. The density function of the envelope is given by

$$p\left(r_i\right) = \frac{r_i}{\sigma^2} e^{\frac{-r_i^2}{2\sigma^2}} \tag{12.1}$$

where r_i is the signal envelope in each branch. If we define two new variables

$$\begin{aligned} \gamma_i &= \frac{\text{Instantaneous signal power in each branch}}{\text{Mean noise power}} \\ \Gamma &= \frac{\text{Mean signal power in each branch}}{\text{Mean noise power}} \end{aligned}$$

then the probability that the C/N is less than or equal to some specified value γ_s is

$$\text{Prob}\left[\gamma_i \leq \gamma_s\right] = 1 - e^{-\gamma_s/\Gamma} \tag{12.2}$$

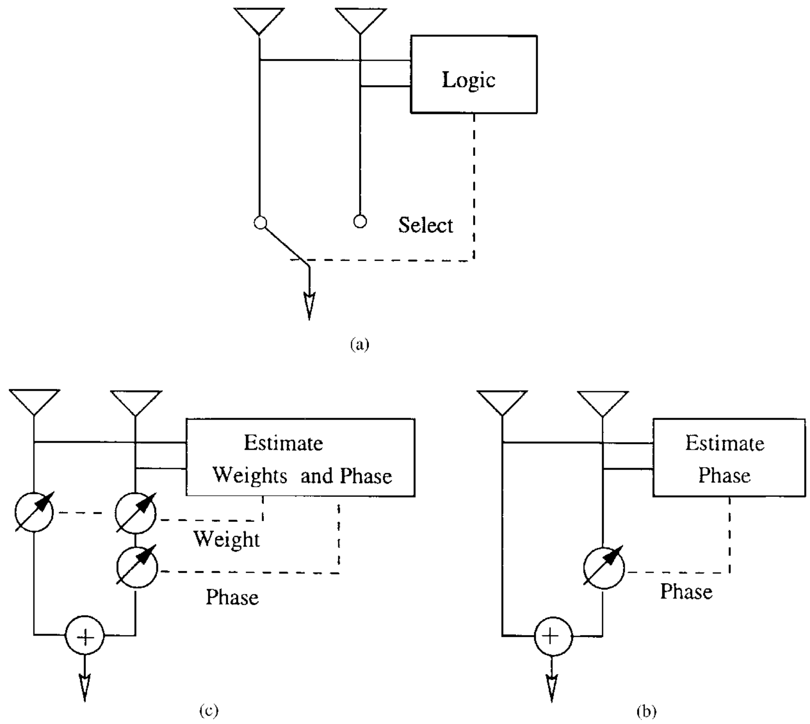

FIGURE 12.2: Diversity combining methods for two diversity branches.

The probability that γ_i in all branches with independent fading will be simultaneously less than or equal to γ_s is then

$$\text{Prob}\left[\gamma_1, \gamma_2, \ldots \gamma_M \leq \gamma_s\right] = \left(1 - e^{-\gamma_s/\Gamma}\right)^M \tag{12.3}$$

This is the distribution of the best signal envelope from the two diversity branches. Figure 12.3 shows the distribution of the combiner output C/N for $M = 1,2,3$, and 4 branches. The improvement in signal quality is significant. For example at 99% reliability level, the improvement in C/N is 10 dB for two branches and 16 dB for four branches.

Selection combining also increases the mean C/N of the combiner output and can be shown to be [3]

$$\text{Mean}\ (\gamma_s) = \Gamma \sum_{k=1}^{M} \frac{1}{k} \tag{12.4}$$

This indicates that with 4 branches, for example, the mean C/N of the selected branch is 2.08 better than the mean C/N in any one branch.

12.3.2 Maximal Ratio Combining

In this technique the M diversity branches are first co-phased and then weighted proportionally to their signal level before summing. See Fig. 12.2(b). The distribution of the maximal ratio combiner

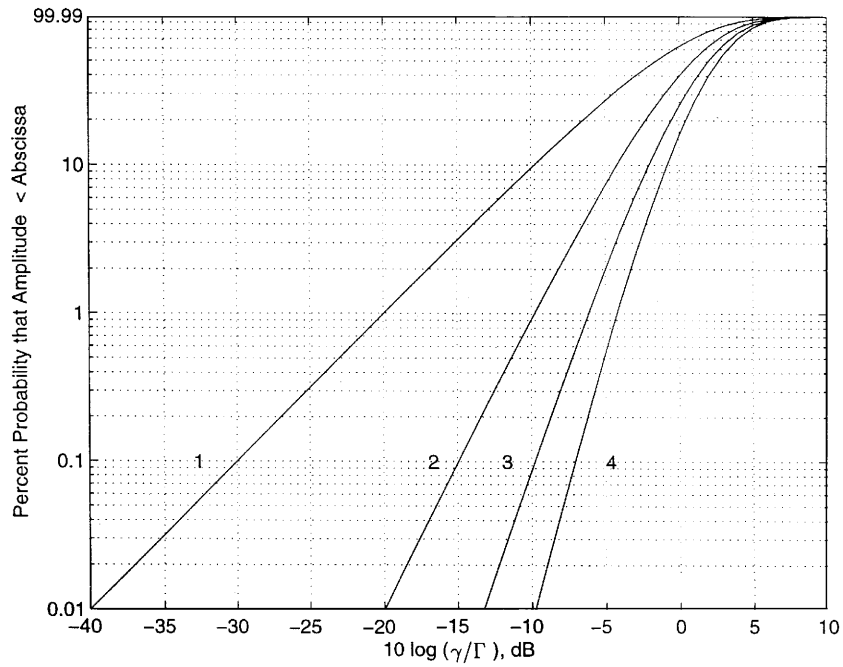

FIGURE 12.3: Probability distribution of signal envelope for selection combining.

has been shown to be [5]

$$\operatorname{Prob}\left[\gamma \leq \gamma_m\right] = 1 - e^{(-\gamma_m/\Gamma)} \sum_{k=1}^{M} \frac{(\gamma_m/\Gamma)^{k-1}}{(k-1)!} \tag{12.5}$$

The distribution of output of a maximal ratio combiner is shown in Fig. 12.4. Maximal ratio combining is known to be optimal in the sense that it yields the best statistical reduction of fading of any linear diversity combiner. In comparison to the selection combiner, at 99% reliability level, the maximal ratio combiner provides a 11.5 dB gain for two branches and a 19 dB gain for four branches, an improvement of 1.5 and 3 dB, respectively, over the selection diversity combiner.

The mean C/N of the combined signal may be easily shown to be

$$\operatorname{Mean}\left(\gamma_m\right) = M\Gamma \tag{12.6}$$

Therefore, combiner output mean varies linearly with M. This confirms the intuitive result that the output C/N averaged over fades should provide gain proportional to the number of diversity branches. This is a situation similar to conventional beamforming.

12.3.3 Equal Gain Combining

In some applications, it may be difficult to estimate the amplitude accurately, the combining gains may all be set to unity, and the diversity branches merely summed after co-phasing. [See Fig. 12.2(c)].

The distribution of equal gain combiner does not have a neat expression and has been computed by numerical evaluation. Its performance has been shown to be very close to within a decibel to

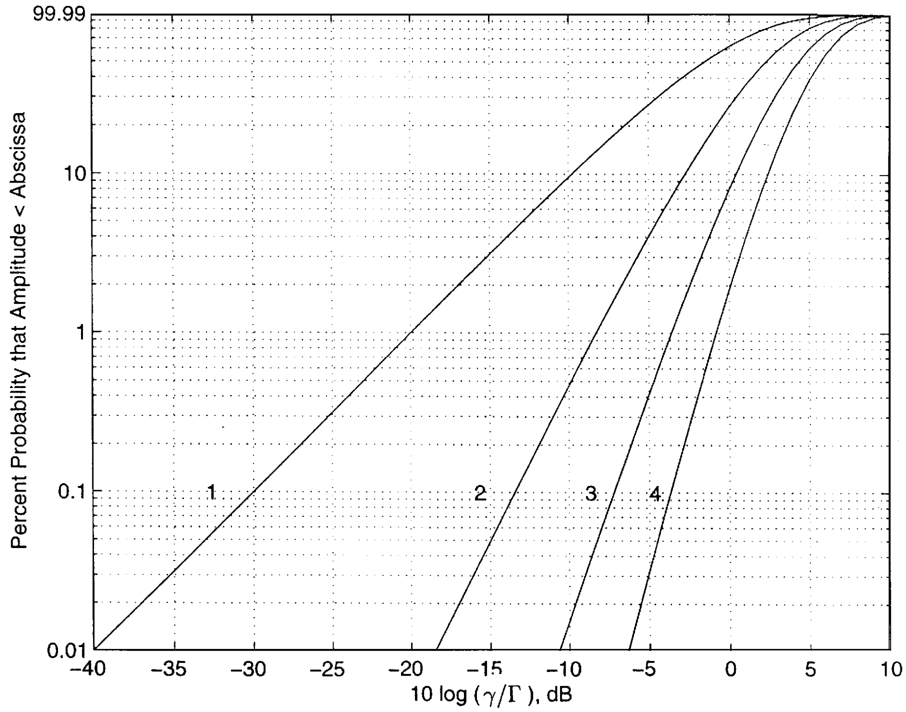

FIGURE 12.4: Probability distribution for signal envelope for maximal ratio combining.

maximal ratio combining. The mean C/N can be shown to be [3]

$$\text{Mean } (\gamma_e) = \Gamma \left[1 + \frac{\pi}{4} (M - 1) \right] \tag{12.7}$$

Like maximal ratio combining, the mean C/N for equal gain combining grows almost linearly with M and is approximately only one decibel poorer than maximal ratio combiner even with an infinite number of branches.

12.3.4 Loss of Diversity Gain Due to Branch Correlation and Unequal Branch Powers

The above analysis assumed that the fading signals in the diversity branches were all uncorrelated and of equal power. In practice, this may be difficult to achieve and as we saw earlier, the branch cross-correlation coefficient $\rho = 0.7$ is considered to be acceptable. Also, equal mean powers in diversity branches are rarely available. In such cases we can expect a certain loss of diversity gain. However, since most of the damage in fading is due to deep fades, and also since the chance of coincidental deep fades is small even for moderate branch correlation, one can expect a reasonable tolerance to branch correlation.

The distribution of the output signal envelope of maximal ratio combiner has been shown to be [6]:

$$\text{Prob}\left[\gamma_m\right] = \sum_{n=1}^{M} \frac{A_n}{2\lambda_n} e^{-\gamma_m/2\lambda_n} \tag{12.8}$$

where λ_n are the eigenvalues of the M $\times$ M branch envelope covariance matrix whose elements are defined by

$$\mathbf{R}_{ij} = E\left[r_i r_j^*\right] \tag{12.9}$$

and A_n is defined by

$$A_n = \prod_{\substack{k=1 \\ k \neq n}}^{M} \frac{1}{1 - \lambda_k / \lambda_n} \tag{12.10}$$

12.4 Effect of Diversity Combining on Bit Error Rate

So far we have studied the distribution of the instantaneous envelope or C/N after diversity combining. We will now briefly survey how diversity combining affects BER performance in digital radio links; we assume maximal ratio combining.

To begin let us first examine the effect of Rayleigh fading on the BER performance of digital transmission links. This has been studied by several authors and is summarized in [7]. Table 12.1 gives the BER expressions in the large E_b/N_0 case for coherent binary PSK and coherent binary orthogonal FSK for unfaded and Rayleigh faded AWGN (additive white Gaussian noise channels) channels. $\bar{E}_b/N_0$ represents the average E_b/N_0 for the fading channel.

TABLE 12.1 Comparison of BER Performance for Unfaded and Rayleigh Faded Signals

Modulaton	Unfaded BER	Faded BER
Coh BPSK	$\frac{1}{2}\operatorname{erfc}\left(\sqrt{E_b/N_0}\right)$	$\frac{1}{4\left(\bar{E}_b/N_0\right)}$
Coh FSK	$\frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{1}{2}E_b/N_0}\right)$	$\frac{1}{2\left(\bar{E}_b/N_0\right)}$

Observe that error rates decrease only inversely with SNR as against exponential decreases for the unfaded channel. Also note that for fading channels, coherent binary PSK is 3 dB better than coherent binary FSK, exactly the same advantage as in unfaded case. Even for modest target BER of 10^{-2} that is usually needed in mobile communications, the loss due to fading can be very high—17.2 dB.

To obtain the BER with maximal ratio diversity combining we have to average the BER expression for the unfaded BER with the distribution obtained for the maximal ratio combiner given in (12.5). Analytical expressions have been derived for these in [7]. For a branch SNR greater than 10 dB, the BER after maximal ratio diversity combining is given in Table 12.2.

We observe that the probability of error varies as $1/\bar{E}_b/N_0$ raised to the Lth power. Thus, diversity reduces the error rate exponentially as the number of independent branches increases.

TABLE 12.2 BER Performance for Coherent BPSK and FSK with Diversity

Modulaton	Post Diversity BER
Coherent BPSK	$\left(\frac{1}{4\,\bar{E}_b/N_0}\right)^L \binom{2L-1}{L}$
Coherent FSK	$\left(\frac{1}{2\bar{E}_b/N_0}\right)^L \binom{2L-1}{L}$

12.5 Concluding Remarks

Diversity provides a powerful technique for combating fading in mobile communication systems. Diversity techniques seek to generate and exploit multiple branches over which the signal shows low fade correlation. To obtain the best diversity performance, the multiple access, modulation, coding and antenna design of the wireless link must all be carefully chosen so as to provide a rich and reliable level of well-balanced, low-correlation diversity branches in the target propagation environment. Successful diversity exploitation can impact a mobile network in several ways. Reduced power requirements can result in increased coverage or improved battery life. Low signal outage improves voice quality and handoff performance. Finally, reduced fade margins directly translate to better reuse factors and, hence, increased system capacity.

Defining Terms

Automatic request for repeat: An error control mechanism in which received packets that cannot be corrected are retransmitted.

Channel coding/Forward error correction: A technique that inserts redundant bits during transmission to help detect and correct bit errors during reception.

Fading: Fluctuation in the signal level due to shadowing and multipath effects.

Frequency hopping: A technique where the signal bursts are transmitted at different frequencies separated by random spacing that are multiples of signal bandwidth.

Interleaving: A form of data scrambling that spreads burst of bit errors evenly over the received data allowing efficient forward error correction.

Outage probability: The probability that the signal level falls below a specified minimum level.

PCS: Personal Communications Services.

RAKE receiver: A receiver used in direct sequence spread spectrum signals. The receiver extracts energy in each path and then adds them together with appropriate weighting and delay.

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