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Digital Communication System Performance[1]

Bernard Sklar
Communications Engineering Services

13.1 Introduction

In this section we examine some fundamental tradeoffs among bandwidth, power, and error performance of digital communication systems. The criteria for choosing modulation and coding schemes, based on whether a system is bandwidth limited or power limited, are reviewed for several system examples. Emphasis is placed on the subtle but straightforward relationships we encounter when transforming from data-bits to channel-bits to symbols to chips.

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The design or definition of any digital communication system begins with a description of the communication link. The *link* is the name given to the communication transmission path from the modulator and transmitter, through the channel, and up to and including the receiver and demodulator. The *channel* is the name given to the propagating medium between the transmitter and receiver. A link description quantifies the average signal power that is received, the available bandwidth, the noise statistics, and other impairments, such as fading. Also needed to define the system are basic requirements, such as the data rate to be supported and the error performance.

13.1.1 The Channel

For radio communications, the concept of *free space* assumes a channel region free of all objects that might affect radio frequency (RF) propagation by absorption, reflection, or refraction. It further assumes that the atmosphere in the channel is perfectly uniform and nonabsorbing, and that the earth is infinitely far away or its reflection coefficient is negligible. The RF energy arriving at the receiver is assumed to be a function of distance from the transmitter (simply following the inverse-square law of optics). In practice, of course, propagation in the atmosphere and near the ground results in refraction, reflection, and absorption, which modify the free space transmission.

13.1.2 The Link

A radio transmitter is characterized by its average output signal power P_t and the gain of its transmitting antenna G_t. The name given to the product $P_t G_t$, with reference to an isotropic antenna is *effective radiated power* (*EIRP*) in watts (or dBW). The predetection average signal power S arriving at the output of the receiver antenna can be described as a function of the *EIRP*, the gain of the receiving antenna G_r, the path loss (or space loss) L_s, and other losses, L_o, as follows [14, 15]:

$$S = \frac{EIRP\ G_r}{L_s L_o} \tag{13.1}$$

The path loss L_s can be written as follows [15]:

$$L_s = \left(\frac{4\pi d}{\lambda}\right)^2 \tag{13.2}$$

where d is the distance between the transmitter and receiver and λ is the wavelength.

We restrict our discussion to those links distorted by the mechanism of additive white Gaussian noise (AWGN) only. Such a noise assumption is a very useful model for a large class of communication systems. A valid approximation for average received noise power N that this model introduces is written as follows [5, 9]:

$$N \cong kT^\circ W \tag{13.3}$$

where k is Boltzmann's constant (1.38×10^{-23} joule/K), T° is effective temperature in kelvin, and W is bandwidth in hertz. Dividing Eq. (13.3) by bandwidth, enables us to write the received noise-power spectral density N_0 as follows:

$$N_0 = \frac{N}{W} = kT^\circ \tag{13.4}$$

Dividing Eq. (13.1) by N_0 yields the received average signal-power to noise-power spectral density S/N_0 as

$$\frac{S}{N_0} = \frac{EIRP\ G_r/T^\circ}{kL_s L_o} \tag{13.5}$$

where G_r/T° is often referred to as the receiver figure of merit. A link budget analysis is a compilation of the power gains and losses throughout the link; it is generally computed in decibels, and thus takes on the bookkeeping appearance of a business enterprise, highlighting the assets and liabilities of the link. Once the value of S/N_0 is specified or calculated from the link parameters, we then shift our attention to optimizing the choice of signalling types for meeting system bandwidth and error performance requirements.

Given the received S/N_0, we can write the received bit-energy to noise-power spectral density E_b/N_0, for any desired data rate R, as follows:

$$\frac{E_b}{N_0} = \frac{ST_b}{N_0} = \frac{S}{N_0}\left(\frac{1}{R}\right) \tag{13.6}$$

Equation (13.6) follows from the basic definitions that received bit energy is equal to received average signal power times the bit duration and that bit rate is the reciprocal of bit duration. Received E_b/N_0 is a key parameter in defining a digital communication system. Its value indicates the apportionment of the received waveform energy among the bits that the waveform represents. At first glance, one might think that a system specification should entail the symbol-energy to noise-power spectral density E_s/N_0 associated with the arriving waveforms. We will show, however, that for a given S/N_0 the value of E_s/N_0 is a function of the modulation and coding. The reason for defining systems in terms of E_b/N_0 stems from the fact that E_b/N_0 depends only on S/N_0 and R and is unaffected by any system design choices, such as modulation and coding.

13.2 Bandwidth and Power Considerations

Two primary communications resources are the received power and the available transmission bandwidth. In many communication systems, one of these resources may be more precious than the other and, hence, most systems can be classified as either bandwidth limited or power limited. In bandwidth-limited systems, spectrally efficient modulation techniques can be used to save bandwidth at the expense of power; in power-limited systems, power efficient modulation techniques can be used to save power at the expense of bandwidth. In both bandwidth- and power-limited systems, error-correction coding (often called channel coding) can be used to save power or to improve error performance at the expense of bandwidth. Recently, trellis-coded modulation (TCM) schemes have been used to improve the error performance of bandwidth-limited channels without any increase in bandwidth [17], but these methods are beyond the scope of this chapter.

13.2.1 The Bandwidth Efficiency Plane

Figure 13.1 shows the abscissa as the ratio of bit-energy to noise-power spectral density E_b/N_0 (in decibels) and the ordinate as the ratio of throughput, R (in bits per second), that can be transmitted per hertz in a given bandwidth W. The ratio R/W is called bandwidth efficiency, since it reflects how efficiently the bandwidth resource is utilized. The plot stems from the Shannon–Hartley capacity theorem [12, 13, 15], which can be stated as

$$C = W \log_2\left(1 + \frac{S}{N}\right) \tag{13.7}$$

where S/N is the ratio of received average signal power to noise power. When the logarithm is taken to the base 2, the capacity C, is given in bits per second. The capacity of a channel defines the maximum number of bits that can be reliably sent per second over the channel. For the case where the data (information) rate R is equal to C, the curve separates a region of practical communication systems from a region where such communication systems cannot operate reliably [12, 15].

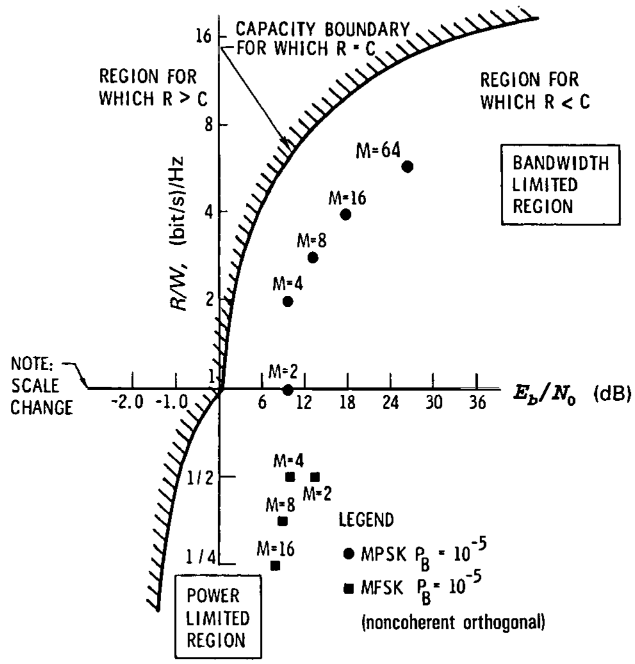

FIGURE 13.1: Bandwidth-efficiency plane.

13.2.2 *M*-ary Signalling

Each symbol in an M-ary alphabet can be related to a unique sequence of m bits, expressed as

$$M = 2^m \qquad \text{or} \qquad m = \log_2 M \tag{13.8}$$

where M is the size of the alphabet. In the case of digital transmission, the term symbol refers to the member of the M-ary alphabet that is transmitted during each symbol duration T_s. To transmit the symbol, it must be mapped onto an electrical voltage or current waveform. Because the waveform represents the symbol, the terms symbol and waveform are sometimes used interchangeably. Since one of M symbols or waveforms is transmitted during each symbol duration T_s, the data rate R in bits per second can be expressed as

$$R = \frac{m}{T_s} = \frac{\log_2 M}{T_s} \tag{13.9}$$

Data-bit-time duration is the reciprocal of data rate. Similarly, symbol-time duration is the reciprocal of symbol rate. Therefore, from Eq. (13.9), we write that the effective time duration T_b of each bit in terms of the symbol duration T_s or the symbol rate R_s is

$$T_b = \frac{1}{R} = \frac{T_s}{m} = \frac{1}{mR_s} \tag{13.10}$$

Then, using Eqs. (13.8) and (13.10) we can express the symbol rate R_s in terms of the bit rate R as follows:

$$R_s = \frac{R}{\log_2 M} \tag{13.11}$$

From Eqs. (13.9) and (13.10), any digital scheme that transmits $m = \log_2 M$ bits in T_s seconds, using a bandwidth of W hertz, operates at a bandwidth efficiency of

$$\frac{R}{W} = \frac{\log_2 M}{WT_s} = \frac{1}{WT_b} \qquad \text{(b/s)/Hz} \tag{13.12}$$

where T_b is the effective time duration of each data bit.

13.2.3 Bandwidth-Limited Systems

From Eq. (13.12), the smaller the WT_b product, the more bandwidth efficient will be any digital communication system. Thus, signals with small WT_b products are often used with bandwidth-limited systems. For example, the European digital mobile telephone system known as Global System for Mobile Communications (GSM) uses Gaussian minimum shift keying (GMSK) modulation having a WT_b product equal to 0.3 Hz/(b/s), where W is the 3-dB bandwidth of a Gaussian filter [4].

For uncoded bandwidth-limited systems, the objective is to maximize the transmitted information rate within the allowable bandwidth, at the expense of E_b/N_0 (while maintaining a specified value of bit-error probability P_B). The operating points for coherent M-ary phase-shift keying (MPSK) at $P_B = 10^{-5}$ are plotted on the bandwidth-efficiency plane of Fig. 13.1. We assume Nyquist (ideal rectangular) filtering at baseband [10]. Thus, for MPSK, the required double-sideband (DSB) bandwidth at an intermediate frequency (IF) is related to the symbol rate as follows:

$$W = \frac{1}{T_s} = R_s \tag{13.13}$$

where T_s is the symbol duration and R_s is the symbol rate. The use of Nyquist filtering results in the minimum required transmission bandwidth that yields zero intersymbol interference; such ideal filtering gives rise to the name Nyquist minimum bandwidth.

From Eqs. (13.12) and (13.13), the bandwidth efficiency of MPSK modulated signals using Nyquist filtering can be expressed as

$$R/W = \log_2 M \qquad \text{(b/s)/Hz} \tag{13.14}$$

The MPSK points in Fig. 13.1 confirm the relationship shown in Eq. (13.14). Note that MPSK modulation is a bandwidth-efficient scheme. As M increases in value, R/W also increases. MPSK modulation can be used for realizing an improvement in bandwidth efficiency at the cost of increased E_b/N_0. Although beyond the scope of this chapter, many highly bandwidth-efficient modulation schemes are under investigation [1].

13.2.4 Power-Limited Systems

Operating points for noncoherent orthogonal M-ary FSK (MFSK) modulation at $P_B = 10^{-5}$ are also plotted on Fig. 13.1. For MFSK, the IF minimum bandwidth is as follows [15]

$$W = \frac{M}{T_s} = M R_s \tag{13.15}$$

where T_s is the symbol duration and R_s is the symbol rate. With MFSK, the required transmission bandwidth is expanded M-fold over binary FSK since there are M different orthogonal waveforms, each requiring a bandwidth of $1/T_s$. Thus, from Eqs. (13.12) and (13.15), the bandwidth efficiency of noncoherent orthogonal MFSK signals can be expressed as

$$\frac{R}{W} = \frac{\log_2 M}{M} \qquad \text{(b/s)/Hz} \tag{13.16}$$

The MFSK points plotted in Fig. 13.1 confirm the relationship shown in Eq. (13.16). Note that MFSK modulation is a bandwidth-expansive scheme. As M increases, R/W decreases. MFSK modulation can be used for realizing a reduction in required E_b/N_0 at the cost of increased bandwidth.

In Eqs. (13.13) and (13.14) for MPSK, and Eqs. (13.15) and (13.16) for MFSK, and for all the points plotted in Fig. 13.1, ideal filtering has been assumed. Such filters are not realizable! For realistic channels and waveforms, the required transmission bandwidth must be increased in order to account for realizable filters.

In the examples that follow, we will consider radio channels that are disturbed only by additive white Gaussian noise (AWGN) and have no other impairments, and for simplicity, we will limit the modulation choice to constant-envelope types, i.e., either MPSK or noncoherent orthogonal MFSK. For an uncoded system, MPSK is selected if the channel is bandwidth limited, and MFSK is selected if the channel is power limited. When error-correction coding is considered, modulation selection is not as simple, because coding techniques can provide power-bandwidth tradeoffs more effectively than would be possible through the use of any M-ary modulation scheme considered in this chapter [3].

In the most general sense, M-ary signalling can be regarded as a waveform-coding procedure, i.e., when we select an M-ary modulation technique instead of a binary one, we in effect have replaced the binary waveforms with better waveforms—either better for bandwidth performance (MPSK) or better for power performance (MFSK). Even though orthogonal MFSK signalling can be thought of as being a coded system, i.e., a first-order Reed-Muller code [8], we restrict our use of the term coded system to those traditional error-correction codes using redundancies, e.g., block codes or convolutional codes.

13.2.5 Minimum Bandwidth Requirements for MPSK and MFSK Signalling

The basic relationship between the symbol (or waveform) transmission rate R_s and the data rate R was shown in Eq. (13.11). Using this relationship together with Eqs. (13.13–13.16) and $R = 9600$ b/s, a summary of symbol rate, minimum bandwidth, and bandwidth efficiency for MPSK and noncoherent orthogonal MFSK was compiled for $M = 2, 4, 8, 16$, and 32 (Table 13.1). Values of E_b/N_0 required to achieve a bit-error probability of 10^{-5} for MPSK and MFSK are also given for each value of M. These entries (which were computed using relationships that are presented later in this chapter) corroborate the tradeoffs shown in Fig. 13.1. As M increases, MPSK signalling provides more bandwidth efficiency at the cost of increased E_b/N_0, whereas MFSK signalling allows for a reduction in E_b/N_0 at the cost of increased bandwidth.

TABLE 13.1 Symbol Rate, Minimum Bandwidth, Bandwidth Efficiency, and Required E_b/N_0 for MPSK and Noncoherent Orthogonal MFSK Signalling at 9600 bit/s

M	m	R (b/s)	R_s (symb/s)	MPSK Minimum Bandwidth (Hz)	MPSK R/W	MPSK E_b/N_0 (dB) $P_B = 10^{-5}$	Noncoherent Orthog MFSK Min Bandwidth (Hz)	MFSK R/W	MFSK E_b/N_0 (dB) $P_B = 10^{-5}$
2	1	9600	9600	9600	1	9.6	19,200	1/2	13.4
4	2	9600	4800	4800	2	9.6	19,200	1/2	10.6
8	3	9600	3200	3200	3	13.0	25,600	3/8	9.1
16	4	9600	2400	2400	4	17.5	38,400	1/4	8.1
32	5	9600	1920	1920	5	22.4	61,440	5/32	7.4

13.3 Example 1: Bandwidth-Limited Uncoded System

Suppose we are given a bandwidth-limited AWGN radio channel with an available bandwidth of $W = 4000$ Hz. Also, suppose that the link constraints (transmitter power, antenna gains, path loss, etc.) result in the ratio of received average signal-power to noise-power spectral density S/N_0 being equal to 53 dB-Hz. Let the required data rate R be equal to 9600 b/s, and let the required bit-error performance P_B be at most 10^{-5}. The goal is to choose a modulation scheme that meets the required performance. In general, an error-correction coding scheme may be needed if none of the allowable modulation schemes can meet the requirements. In this example, however, we shall find that the use of error-correction coding is not necessary.

13.3.1 Solution to Example 1

For any digital communication system, the relationship between received S/N_0 and received bit-energy to noise-power spectral density, E_b/N_0 was given in Eq. (13.6) and is briefly rewritten as

$$\frac{S}{N_0} = \frac{E_b}{N_0} R \tag{13.17}$$

Solving for E_b/N_0 in decibels, we obtain

$$\begin{aligned} \frac{E_b}{N_0}\,(\text{dB}) &= \frac{S}{N_0}\,(\text{dB-Hz}) - R\,(\text{dB-b/s}) \\ &= 53\ \text{dB-Hz} - \left(10 \times \log_{10} 9600\right)\ \text{dB-b/s} \\ &= 13.2\ \text{dB (or 20.89)} \end{aligned} \tag{13.18}$$

Since the required data rate of 9600 b/s is much larger than the available bandwidth of 4000 Hz, the channel is bandwidth limited. We therefore select MPSK as our modulation scheme. We have confined the possible modulation choices to be constant-envelope types; without such a restriction, we would be able to select a modulation type with greater bandwidth efficiency. To conserve power, we compute the *smallest possible* value of M such that the MPSK minimum bandwidth does not exceed the available bandwidth of 4000 Hz. Table 13.1 shows that the smallest value of M meeting this requirement is $M = 8$. Next we determine whether the required bit-error performance of $P_B \leq 10^{-5}$ can be met by using 8-PSK modulation alone or whether it is necessary to use an error-correction coding scheme. Table 13.1 shows that 8-PSK alone will meet the requirements, since the required E_b/N_0 listed for 8-PSK is less than the received E_b/N_0 derived in Eq. (13.18). Let us imagine that we do not have Table 13.1, however, and evaluate whether or not error-correction coding is necessary.

Figure 13.2 shows the basic modulator/demodulator (MODEM) block diagram summarizing the functional details of this design. At the modulator, the transformation from data bits to symbols yields an output symbol rate R_s, that is, a factor $\log_2 M$ smaller than the input data-bit rate R, as is seen in Eq. (13.11). Similarly, at the input to the demodulator, the symbol-energy to noise-power spectral density E_S/N_0 is a factor $\log_2 M$ larger than E_b/N_0, since each symbol is made up of $\log_2 M$ bits. Because E_S/N_0 is larger than E_b/N_0 by the same factor that R_s is smaller than R, we can expand Eq. (13.17), as follows:

$$\frac{S}{N_0} = \frac{E_b}{N_0} R = \frac{E_s}{N_0} R_s \tag{13.19}$$

The demodulator receives a waveform (in this example, one of $M = 8$ possible phase shifts) during each time interval T_s. The probability that the demodulator makes a symbol error $P_E(M)$ is well approximated by the following equation for $M > 2$ [6]:

$$P_E(M) \cong 2Q\left[\sqrt{\frac{2E_s}{N_0}} \sin\left(\frac{\pi}{M}\right)\right] \tag{13.20}$$

where $Q(x)$, sometimes called the complementary error function, represents the probability under the tail of a zero-mean unit-variance Gaussian density function. It is defined as follows [18]:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{u^2}{2}\right) \mathrm{d}u \tag{13.21}$$

A good approximation for $Q(x)$, valid for $x > 3$, is given by the following equation [2]

$$Q(x) \cong \frac{1}{x\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \tag{13.22}$$

In Fig. 13.2 and all of the figures that follow, rather than show explicit probability relationships, the generalized notation $f(x)$ has been used to indicate some functional dependence on x.

A traditional way of characterizing communication efficiency in digital systems is in terms of the received E_b/N_0 in decibels. This E_b/N_0 description has become standard practice, but recall that there are no bits at the input to the demodulator; there are only waveforms that have been assigned bit meanings. The received E_b/N_0 represents a bit-apportionment of the arriving waveform energy.

To solve for $P_E(M)$ in Eq. (13.20), we first need to compute the ratio of received symbol-energy to noise-power spectral density E_s/N_0. Since from Eq. (13.18)

$$\frac{E_b}{N_0} = 13.2 \text{ dB (or 20.89)}$$

and because each symbol is made up of $\log_2 M$ bits, we compute the following using $M = 8$.

$$\frac{E_s}{N_0} = \left(\log_2 M\right) \frac{E_b}{N_0} = 3 \times 20.89 = 62.67 \tag{13.23}$$

Using the results of Eq. (13.23) in Eq. (13.20), yields the symbol-error probability $P_E = 2.2 \times 10^{-5}$. To transform this to bit-error probability, we use the relationship between bit-error probability P_B and symbol-error probability P_E, for multiple-phase signalling [8] for $P_E \ll 1$ as follows:

$$P_B \cong \frac{P_E}{\log_2 M} = \frac{P_E}{m} \tag{13.24}$$

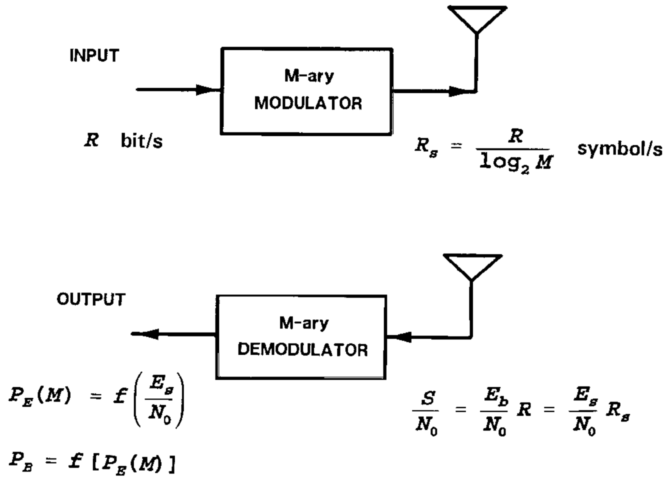

FIGURE 13.2: Basic modulator/demodulator (MODEM) without channel coding.

which is a good approximation when Gray coding is used for the bit-to-symbol assignment [6]. This last computation yields $P_B = 7.3 \times 10^{-6}$, which meets the required bit-error performance. No error-correction coding is necessary, and 8-PSK modulation represents the design choice to meet the requirements of the bandwidth-limited channel, which we had predicted by examining the required E_b/N_0 values in Table 13.1.

13.4 Example 2: Power-Limited Uncoded System

Now, suppose that we have exactly the same data rate and bit-error probability requirements as in Example 1, but let the available bandwidth W be equal to 45 kHz, and the available S/N_0 be equal to 48 dB-Hz. The goal is to choose a modulation or modulation/coding scheme that yields the required performance. We shall again find that error-correction coding is not required.

13.4.1 Solution to Example 2

The channel is clearly not bandwidth limited since the available bandwidth of 45 kHz is more than adequate for supporting the required data rate of 9600 bit/s. We find the received E_b/N_0 from Eq. (13.18), as follows:

$$\frac{E_b}{N_0}\ (\text{dB}) = 48\ \text{dB-Hz} - \left(10 \times \log_{10} 9600\right)\ \text{dB-b/s} = 8.2\ \text{dB (or 6.61)} \tag{13.25}$$

Since there is abundant bandwidth but a relatively small E_b/N_0 for the required bit-error probability, we consider that this channel is power limited and choose MFSK as the modulation scheme. To conserve power, we search for the *largest possible* M such that the MFSK minimum bandwidth is not expanded beyond our available bandwidth of 45 kHz. A search results in the choice of $M = 16$ (Table 13.1). Next, we determine whether the required error performance of $P_B \le 10^{-5}$ can be met by using 16-FSK alone, i.e., without error-correction coding. Table 13.1 shows that 16-FSK alone meets the requirements, since the required E_b/N_0 listed for 16-FSK is less than the received E_b/N_0

derived in Eq. (13.25). Let us imagine again that we do not have Table 13.1, and evaluate whether or not error-correction coding is necessary.

The block diagram in Fig. 13.2 summarizes the relationships between symbol rate R_s, and bit rate R, and between E_s/N_0 and E_b/N_0, which is identical to each of the respective relationships in Example 1. The 16-FSK demodulator receives a waveform (one of 16 possible frequencies) during each symbol time interval T_s. For noncoherent orthogonal MFSK, the probability that the demodulator makes a symbol error $P_E(M)$ is approximated by the following upper bound [20]:

$$P_E(M) \leq \frac{M-1}{2} \exp\left(-\frac{E_s}{2N_0}\right) \tag{13.26}$$

To solve for $P_E(M)$ in Eq. (13.26), we compute E_S/N_0, as in Example 1. Using the results of Eq. (13.25) in Eq. (13.23), with $M = 16$, we get

$$\frac{E_s}{N_0} = \left(\log_2 M\right) \frac{E_b}{N_0} = 4 \times 6.61 = 26.44 \tag{13.27}$$

Next, using the results of Eq. (13.27) in Eq. (13.26), yields the symbol-error probability $P_E = 1.4 \times 10^{-5}$. To transform this to bit-error probability, P_B, we use the relationship between P_B and P_E for orthogonal signalling [20], given by

$$P_B = \frac{2^{m-1}}{(2^m - 1)} P_E \tag{13.28}$$

This last computation yields $P_B = 7.3 \times 10^{-6}$, which meets the required bit-error performance. Thus, we can meet the given specifications for this power-limited channel by using 16-FSK modulation, without any need for error-correction coding, as we had predicted by examining the required E_b/N_0 values in Table 13.1.

13.5 Example 3: Bandwidth-Limited and Power-Limited Coded System

We start with the same channel parameters as in Example 1 ($W = 4000$ Hz, $S/N_0 = 53$ dB-Hz, and $R = 9600$ b/s), with one exception.

In this example, we specify that P_B must be at most 10^{-9}. Table 13.1 shows that the system is both bandwidth limited and power limited, based on the available bandwidth of 4000 Hz and the available E_b/N_0 of 20.2 dB, from Eq. (13.18); 8-PSK is the only possible choice to meet the bandwidth constraint; however, the available E_b/N_0 of 20.2 dB is certainly insufficient to meet the required P_B of 10^{-9}. For this small value of P_B, we need to consider the performance improvement that error-correction coding can provide within the available bandwidth. In general, one can use convolutional codes or block codes.

The Bose–Chaudhuri–Hocquenghem (BCH) codes form a large class of powerful error-correcting cyclic (block) codes [7]. To simplify the explanation, we shall choose a block code from the BCH family. Table 13.2 presents a partial catalog of the available BCH codes in terms of n, k, and t, where k represents the number of information (or data) bits that the code transforms into a longer block of n coded bits (or channel bits), and t represents the largest number of incorrect channel bits that the code can correct within each n-sized block. The rate of a code is defined as the ratio k/n; its inverse represents a measure of the code's redundancy [7].

TABLE 13.2
BCH Codes (Partial Catalog)

n	k	t
7	4	1
15	11	1
	7	2
	5	3
31	26	1
	21	2
	16	3
	11	5
63	57	1
	51	2
	45	3
	39	4
	36	5
	30	6
127	120	1
	113	2
	106	3
	99	4
	92	5
	85	6
	78	7
	71	9
	64	10

13.5.1 Solution to Example 3

Since this example has the same bandwidth-limited parameters given in Example 1, we start with the same 8-PSK modulation used to meet the stated bandwidth constraint. We now employ error-correction coding, however, so that the bit-error probability can be lowered to $P_B \leq 10^{-9}$.

To make the optimum code selection from Table 13.2, we are guided by the following goals.

1. The output bit-error probability of the combined modulation/coding system must meet the system error requirement.
2. The rate of the code must not expand the required transmission bandwidth beyond the available channel bandwidth.
3. The code should be as simple as possible. Generally, the shorter the code, the simpler will be its implementation.

The uncoded 8-PSK minimum bandwidth requirement is 3200 Hz (Table 13.1) and the allowable channel bandwidth is 4000 Hz, and so the uncoded signal bandwidth can be increased by no more than a factor of 1.25 (i.e., an expansion of 25%). The very first step in this (simplified) code selection example is to eliminate the candidates in Table 13.2 that would expand the bandwidth by more than 25%. The remaining entries form a much reduced set of bandwidth-compatible codes (Table 13.3).

In Table 13.3, a column designated Coding Gain G (for MPSK at $P_B = 10^{-9}$) has been added. Coding gain in decibels is defined as follows:

$$G = \left(\frac{E_b}{N_0}\right)_{\text{uncoded}} - \left(\frac{E_b}{N_0} s\right)_{\text{coded}} \tag{13.29}$$

G can be described as the reduction in the required E_b/N_0 (in decibels) that is needed due to the error-performance properties of the channel coding. G is a function of the modulation type and bit-error probability, and it has been computed for MPSK at $P_B = 10^{-9}$ (Table 13.3). For MPSK

TABLE 13.3
Bandwidth-Compatible BCH Codes

n	k	t	Coding Gain, G (dB) MPSK, $P_B = 10^{-9}$
31	26	1	2.0
63	57	1	2.2
	51	2	3.1
127	120	1	2.2
	113	2	3.3
	106	3	3.9

modulation, G is relatively independent of the value of M. Thus, for a particular bit-error probability, a given code will provide about the same coding gain when used with any of the MPSK modulation schemes. Coding gains were calculated using a procedure outlined in the subsequent Calculating Coding Gain section.

A block diagram summarizes this system, which contains both modulation and coding (Fig. 13.3). The introduction of encoder/decoder blocks brings about additional transformations. The relationships that exist when transforming from R b/s to R_c channel-b/s to R_s symbol/s are shown at the encoder/modulator. Regarding the channel-bit rate R_c, some authors prefer to use the units of channel-symbol/s (or code-symbol/s). The benefit is that error-correction coding is often described more efficiently with nonbinary digits. We reserve the term symbol for that group of bits mapped onto an electrical waveform for transmission, and we designate the units of R_c to be channel-b/s (or coded-b/s).

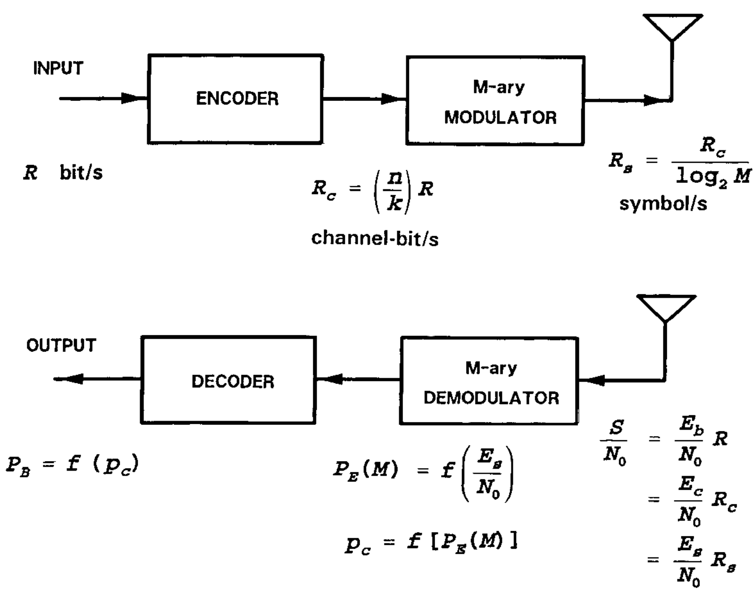

FIGURE 13.3: MODEM with channel coding.

We assume that our communication system cannot tolerate any message delay, so that the channel-bit rate R_c must exceed the data-bit rate R by the factor n/k. Further, each symbol is made up of $\log_2 M$ channel bits, and so the symbol rate R_s is less than R_c by the factor $\log_2 M$. For a system containing both modulation and coding, we summarize the rate transformations as follows:

$$R_c = \left(\frac{n}{k}\right) R \tag{13.30}$$

$$R_s = \frac{R_c}{\log_2 M} \tag{13.31}$$

At the demodulator/decoder in Fig.13.3, the transformations among data-bit energy, channel- bit energy, and symbol energy are related (in a reciprocal fashion) by the same factors as shown among the rate transformations in Eqs. (13.30) and (13.31). Since the encoding transformation has replaced k data bits with n channel bits, then the ratio of channel-bit energy to noise-power spectral density E_c/N_0 is computed by decrementing the value of E_b/N_0 by the factor k/n. Also, since each transmission symbol is made up of $\log_2 M$ channel bits, then E_S/N_0, which is needed in Eq. (13.20) to solve for P_E, is computed by incrementing E_c/N_0 by the factor $\log_2 M$. For a system containing both modulation and coding, we summarize the energy to noise-power spectral density transformations as follows:

$$\frac{E_c}{N_0} = \left(\frac{k}{n}\right) \frac{E_b}{N_0} \tag{13.32}$$

$$\frac{E_s}{N_0} = \left(\log_2 M\right) \frac{E_c}{N_0} \tag{13.33}$$

Using Eqs. (13.30) and (13.31), we can now expand the expression for S/N_0 in Eq. (13.19), as follows (Appendix).

$$\frac{S}{N_0} = \frac{E_b}{N_0} R = \frac{E_c}{N_0} R_c = \frac{E_s}{N_0} R_s \tag{13.34}$$

As before, a standard way of describing the link is in terms of the received E_b/N_0 in decibels. However, there are no data bits at the input to the demodulator, and there are no channel bits; there are only waveforms that have bit meanings and, thus, the waveforms can be described in terms of bit-energy apportionments.

Since S/N_0 and R were given as 53 dB-Hz and 9600 b/s, respectively, we find as before, from Eq. (13.18), that the received $E_b/N_0 = 13.2$ dB. The received E_b/N_0 is fixed and independent of n, k, and t (Appendix). As we search, in Table 13.3 for the ideal code to meet the specifications, we can iteratively repeat the computations suggested in Fig. 13.3. It might be useful to program on a personal computer (or calculator) the following four steps as a function of n, k, and t. Step 1 starts by combining Eqs. (13.32) and (13.33), as follows.

Step 1:

$$\frac{E_s}{N_0} = \left(\log_2 M\right) \frac{E_c}{N_0} = \left(\log_2 M\right) \left(\frac{k}{n}\right) \frac{E_b}{N_0} \tag{13.35}$$

Step 2:

$$P_E(M) \cong 2Q\left[\sqrt{\frac{2E_s}{N_0}} \sin\left(\frac{\pi}{M}\right)\right] \tag{13.36}$$

which is the approximation for symbol-error probability P_E rewritten from Eq. (13.20). At each symbol-time interval, the demodulator makes a symbol decision, but it delivers a channel-bit sequence representing that symbol to the decoder. When the channel-bit output of the demodulator is quantized to two levels, 1 and 0, the demodulator is said to make hard decisions. When the output is quantized to more than two levels, the demodulator is said to make soft decisions [15]. Throughout this paper, we shall assume hard-decision demodulation.

Now that we have a decoder block in the system, we designate the channel-bit-error probability out of the demodulator and into the decoder as p_c, and we reserve the notation P_B for the bit-error probability out of the decoder. We rewrite Eq. (13.24) in terms of p_c for $P_E \ll 1$ as follows.

Step 3:

$$p_c \cong \frac{P_E}{\log_2 M} = \frac{P_E}{m} \tag{13.37}$$

relating the channel-bit-error probability to the symbol-error probability out of the demodulator, assuming Gray coding, as referenced in Eq. (13.24).

For traditional channel-coding schemes and a given value of received S/N_0, the value of E_s/N_0 with coding will always be less than the value of E_s/N_0 without coding. Since the demodulator with coding receives less E_s/N_0, it makes more errors! When coding is used, however, the system error-performance does not only depend on the performance of the demodulator, it also depends on the performance of the decoder. For error-performance improvement due to coding, the decoder must provide enough error correction to more than compensate for the poor performance of the demodulator.

The final output decoded bit-error probability P_B depends on the particular code, the decoder, and the channel-bit-error probability p_c. It can be expressed by the following approximation [11].

Step 4:

$$P_B \cong \frac{1}{n} \sum_{j=t+1}^{n} j \binom{n}{j} p_c^j \,(1 - p_c)^{n-j} \tag{13.38}$$

where t is the largest number of channel bits that the code can correct within each block of n bits. Using Eqs. (13.35–13.38) in the four steps, we can compute the decoded bit-error probability P_B as a function of n, k, and t for each of the codes listed in Table 13.3. The entry that meets the stated error requirement with the largest possible code rate and the smallest value of n is the double-error correcting (63, 51) code. The computations are as follows.

Step 1:

$$\frac{E_s}{N_0} = 3\left(\frac{51}{63}\right) 20.89 = 50.73$$

where $M = 8$, and the received $E_b/N_0 = 13.2$ dB (or 20.89).

Step 2:

$$P_E \cong 2Q\left[\sqrt{101.5} \times \sin\left(\frac{\pi}{8}\right)\right] = 2Q(3.86) = 1.2 \times 10^{-4}$$

Step 3:

$$p_c \cong \frac{1.2 \times 10^{-4}}{3} = 4 \times 10^{-5}$$

Step 4:

$$\begin{aligned} P_B &\cong \frac{3}{63}\binom{63}{3}\left(4 \times 10^{-5}\right)^3 \left(1 - 4 \times 10^{-5}\right)^{60} \\ &\quad + \frac{4}{63}\binom{63}{4}\left(4 \times 10^{-5}\right)^4 \left(1 - 4 \times 10^{-5}\right)^{59} + \cdots \\ &= 1.2 \times 10^{-10} \end{aligned}$$

where the bit-error-correcting capability of the code is $t = 2$. For the computation of P_B in step 4, we need only consider the first two terms in the summation of Eq. (13.38) since the other terms have a vanishingly small effect on the result. Now that we have selected the (63, 51) code, we can compute the values of channel-bit rate R_c and symbol rate R_s using Eqs. (13.30) and (13.31), with $M = 8$,

$$\begin{aligned} R_c &= \left(\frac{n}{k}\right) R = \left(\frac{63}{51}\right) 9600 \approx 11{,}859 \text{ channel-b/s} \\ R_s &= \frac{R_c}{\log_2 M} = \frac{11859}{3} = 3953 \text{ symbol/s} \end{aligned}$$

13.5.2 Calculating Coding Gain

Perhaps a more direct way of finding the simplest code that meets the specified error performance is to first compute how much coding gain G is required in order to yield $P_B = 10^{-9}$ when using 8-PSK modulation alone; then, from Table 13.3, we can simply choose the code that provides this performance improvement. First, we find the uncoded E_s/N_0 that yields an error probability of $P_B = 10^{-9}$, by writing from Eqs. (13.24) and (13.36), the following:

$$P_B \cong \frac{P_E}{\log_2 M} \cong \frac{2Q\left[\sqrt{\frac{2E_s}{N_0}} \sin\left(\frac{\pi}{M}\right)\right]}{\log_2 M} = 10^{-9} \tag{13.39}$$

At this low value of bit-error probability, it is valid to use Eq. (13.22) to approximate $Q(x)$ in Eq. (13.39) By trial and error (on a programmable calculator), we find that the uncoded $E_s/N_0 = 120.67 = 20.8$ dB, and since each symbol is made up of $\log_2 8 = 3$ bits, the required $(E_b/N_0)_{\text{uncoded}} = 120.67/3 = 40.22 = 16$ dB. From the given parameters and Eq. (13.18), we know that the received $(E_b/N_0)_{\text{coded}} = 13.2$ dB. Using Eq. (13.29), the required coding gain to meet the bit-error performance of $P_B = 10^{-9}$ in decibels is

$$G = \left(\frac{E_b}{N_0}\right)_{\text{uncoded}} - \left(\frac{E_b}{N_0}\right)_{\text{coded}} = 16 - 13.2 = 2.8$$

To be precise, each of the E_b/N_0 values in the preceding computation must correspond to exactly the same value of bit-error probability (which they do not). They correspond to $P_B = 10^{-9}$ and $P_B = 1.2 \times 10^{-10}$, respectively. At these low probability values, however, even with such a discrepancy, this computation still provides a good approximation of the required coding gain. In searching Table 13.3 for the simplest code that will yield a coding gain of at least 2.8 dB, we see that the choice is the (63, 51) code, which corresponds to the same code choice that we made earlier.

13.6 Example 4: Direct-Sequence (DS) Spread-Spectrum Coded System

Spread-spectrum systems are not usually classified as being bandwidth- or power-limited. They are generally perceived to be power-limited systems, however, because the bandwidth occupancy of the information is much larger than the bandwidth that is intrinsically needed for the information transmission. In a direct-sequence spread-spectrum (DS/SS) system, spreading the signal bandwidth by some factor permits lowering the signal-power spectral density by the same factor (the total average signal power is the same as before spreading). The bandwidth spreading is typically accomplished by multiplying a relatively narrowband data signal by a wideband spreading signal. The spreading signal or spreading code is often referred to as a pseudorandom code or PN code.

13.6.1 Processing Gain

A typical DS/SS radio system is often described as a two-step BPSK modulation process. In the first step, the carrier wave is modulated by a bipolar data waveform having a value $+1$ or -1 during each data-bit duration; in the second step, the output of the first step is multiplied (modulated) by a bipolar PN-code waveform having a value $+1$ or -1 during each PN-code-bit duration. In reality, DS/SS systems are usually implemented by first multiplying the data waveform by the PN-code waveform and then making a single pass through a BPSK modulator. For this example, however, it is useful to characterize the modulation process in two separate steps—the outer modulator/demodulator for the data, and the inner modulator/demodulator for the PN code (Fig. 13.4).

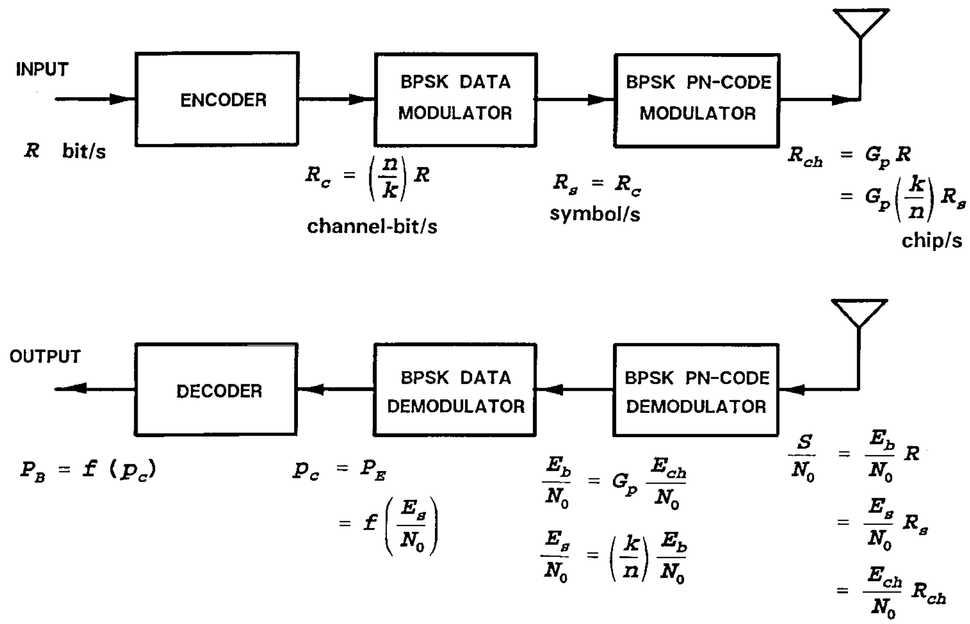

FIGURE 13.4: Direct-sequence spread-spectrum MODEM with channel coding.

A spread-spectrum system is characterized by a processing gain G_p, that is defined in terms of the spread-spectrum bandwidth W_{ss} and the data rate R as follows [20]:

$$G_p = \frac{W_{ss}}{R} \tag{13.40}$$

For a DS/SS system, the PN-code bit has been given the name chip, and the spread-spectrum signal bandwidth can be shown to be about equal to the chip rate R_{ch} as follows:

$$G_p = \frac{R_{ch}}{R} \tag{13.41}$$

Some authors define processing gain to be the ratio of the spread-spectrum bandwidth to the symbol rate. This definition separates the system performance that is due to bandwidth spreading from the performance that is due to error-correction coding. Since we ultimately want to relate all of the coding mechanisms relative to the information source, we shall conform to the most usually accepted definition for processing gain, as expressed in Eqs. (13.40) and (13.41).

A spread-spectrum system can be used for interference rejection and for multiple access (allowing multiple users to access a communications resource simultaneously). The benefits of DS/SS signals are best achieved when the processing gain is very large; in other words, the chip rate of the spreading (or PN) code is much larger than the data rate. In such systems, the large value of G_p allows the signalling chips to be transmitted at a power level well below that of the thermal noise. We will use a value of $G_p = 1000$. At the receiver, the despreading operation correlates the incoming signal with a synchronized copy of the PN code and, thus, accumulates the energy from multiple (G_p) chips to yield the energy per data bit. The value of G_p has a major influence on the performance of the spread-spectrum system application. We shall see, however, that the value of G_p has no effect on the received E_b/N_0. In other words, spread spectrum techniques offer no error-performance advantage over thermal noise. For DS/SS systems, there is no disadvantage either! Sometimes such spread-spectrum radio systems are employed only to enable the transmission of very small power-spectral densities and thus avoid the need for FCC licensing [16].

13.6.2 Channel Parameters for Example 13.4

Consider a DS/SS radio system that uses the same (63, 51) code as in the previous example. Instead of using MPSK for the data modulation, we shall use BPSK. Also, we shall use BPSK for modulating the PN-code chips. Let the received $S/N_0 = 48$ dB-Hz, the data rate $R = 9600$ b/s, and the required $P_B \leq 10^{-6}$. For simplicity, assume that there are no bandwidth constraints. Our task is simply to determine whether or not the required error performance can be achieved using the given system architecture and design parameters. In evaluating the system, we will use the same type of transformations used in the previous examples.

13.6.3 Solution to Example 13.4

A typical DS/SS system can be implemented more simply than the one shown in Fig. 13.4. The data and the PN code would be combined at baseband, followed by a single pass through a BPSK modulator. We will, however, assume the existence of the individual blocks in Fig. 13.4 because they enhance our understanding of the transformation process. The relationships in transforming from data bits, to channel bits, to symbols, and to chips Fig. 13.4 have the same pattern of subtle but straightforward transformations in rates and energies as previous relationships (Figs. 13.2 and 13.3). The values of R_c, R_s, and R_{ch} can now be calculated immediately since the (63, 51) BCH code has already been selected. From Eq. (13.30) we write

$$R_c = \left(\frac{n}{k}\right) R = \left(\frac{63}{51}\right) 9600 \approx 11{,}859 \text{ channel-b/s}$$

Since the data modulation considered here is BPSK, then from Eq. (13.31) we write

$$R_s = R_c \approx 11{,}859 \text{ symbol/s}$$

and from Eq. (13.41), with an assumed value of $G_p = 1000$

$$R_{\text{ch}} = G_p R = 1000 \times 9600 = 9.6 \times 10^6 \text{ chip/s}$$

Since we have been given the same S/N_0 and the same data rate as in Example 2, we find the value of received E_b/N_0 from Eq. (13.25) to be 8.2 dB (or 6.61). At the demodulator, we can now expand the expression for S/N_0 in Eq. (13.34) and the Appendix as follows:

$$\frac{S}{N_0} = \frac{E_b}{N_0} R = \frac{E_c}{N_0} R_c = \frac{E_s}{N_0} R_s = \frac{E_{\text{ch}}}{N_0} R_{\text{ch}} \tag{13.42}$$

Corresponding to each transformed entity (data bit, channel bit, symbol, or chip) there is a change in rate and, similarly, a reciprocal change in energy-to-noise spectral density for that received entity. Equation (13.42) is valid for any such transformation when the rate and energy are modified in a reciprocal way. There is a kind of *conservation of power* (or energy) phenomenon that exists in the transformations. The total received average power (or total received energy per symbol duration) is fixed regardless of how it is computed, on the basis of data bits, channel bits, symbols, or chips.

The ratio E_{ch}/N_0 is much lower in value than E_b/N_0. This can be seen from Eqs. (13.42) and (13.41), as follows:

$$\frac{E_{\text{ch}}}{N_0} = \frac{S}{N_0}\left(\frac{1}{R_{\text{ch}}}\right) = \frac{S}{N_0}\left(\frac{1}{G_p R}\right) = \left(\frac{1}{G_p}\right)\frac{E_b}{N_0} \tag{13.43}$$

But, even so, the despreading function (when properly synchronized) accumulates the energy contained in a quantity G_p of the chips, yielding the same value $E_b/N_0 = 8.2$ dB, as was computed earlier from Eq. (13.25). Thus, the DS spreading transformation has no effect on the error performance of an AWGN channel [15], and the value of G_p has no bearing on the value of P_B in this example.

From Eq. (13.43), we can compute, in decibels,

$$\begin{aligned} \frac{E_{\text{ch}}}{N_0} &= E_b/N_0 - G_p \\ &= 8.2 - (10 \times \log_{10} 1000) \\ &= -21.8 \end{aligned} \tag{13.44}$$

The chosen value of processing gain ($G_p = 1000$) enables the DS/SS system to operate at a value of chip energy well below the thermal noise, with the same error performance as without spreading.

Since BPSK is the data modulation selected in this example, each message symbol therefore corresponds to a single channel bit, and we can write

$$\frac{E_s}{N_0} = \frac{E_c}{N_0} = \left(\frac{k}{n}\right)\frac{E_b}{N_0} = \left(\frac{51}{63}\right) \times 6.61 = 5.35 \tag{13.45}$$

where the received $E_b/N_0 = 8.2$ dB (or 6.61). Out of the BPSK data demodulator, the symbol-error probability P_E (and the channel-bit error probability p_c) is computed as follows [15]:

$$p_c = P_E = Q\left(\sqrt{\frac{2E_c}{N_0}}\right) \tag{13.46}$$

Using the results of Eq. (13.45) in Eq. (13.46) yields

$$p_c = Q(3.27) = 5.8 \times 10^{-4}$$

Finally, using this value of p_c in Eq. (13.38) for the (63,51) double-error correcting code yields the output bit-error probability of $P_B = 3.6 \times 10^{-7}$. We can, therefore, verify that for the given architecture and design parameters of this example the system does, in fact, achieve the required error performance.

13.7 Conclusion

The goal of this section has been to review fundamental relationships used in evaluating the performance of digital communication systems. First, we described the concept of a link and a channel and examined a radio system from its transmitting segment up through the output of the receiving antenna. We then examined the concept of bandwidth-limited and power-limited systems and how such conditions influence the system design when the choices are confined to MPSK and MFSK modulation. Most important, we focused on the definitions and computations involved in transforming from data bits to channel bits to symbols to chips. In general, most digital communication systems share these concepts; thus, understanding them should enable one to evaluate other such systems in a similar way.

Appendix: Received E_b/N_0 Is Independent of the Code Parameters

Starting with the basic concept that the received average signal power S is equal to the received symbol or waveform energy, E_s, divided by the symbol-time duration, T_s (or multiplied by the symbol rate, R_s), we write

$$\frac{S}{N_0} = \frac{E_s/T_s}{N_0} = \frac{E_s}{N_0} R_s \tag{A13.1}$$

where N_0 is noise-power spectral density.

Using Eqs. (13.27) and (13.25), rewritten as

$$\frac{E_s}{N_0} = \left(\log_2 M\right) \frac{E_c}{N_0} \qquad \text{and} \qquad R_s = \frac{R_c}{\log_2 M}$$

let us make substitutions into Eq. (A13.1), which yields

$$\frac{S}{N_0} = \frac{E_c}{N_0} R_c \tag{A13.2}$$

Next, using Eqs. (13.26) and (13.24), rewritten as

$$\frac{E_c}{N_0} = \left(\frac{k}{n}\right) \frac{E_b}{N_0} \qquad \text{and} \qquad R_c = \left(\frac{n}{k}\right) R$$

let us now make substitutions into Eq. (A13.2), which yields the relationship expressed in Eq. (13.11)

$$\frac{S}{N_0} = \frac{E_b}{N_0} R \tag{A13.3}$$

Hence, the received E_b/N_0 is only a function of the received S/N_0 and the data rate R. It is independent of the code parameters, n, k, and t. These results are summarized in Fig. 13.3.

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Further Information

A useful compilation of selected papers can be found in: *Cellular Radio & Personal Communications–A Book of Selected Readings*, edited by Theodore S. Rappaport, Institute of Electrical and Electronics Engineers, Inc., Piscataway, New Jersey, 1995. Fundamental design issues, such as propagation, modulation, channel coding, speech coding, multiple-accessing and networking, are well represented in this volume.

Another useful sourcebook that covers the fundamentals of mobile communications in great detail is: *Mobile Radio Communications*, edited by Raymond Steele, Pentech Press, London 1992. This volume is also available through the Institute of Electrical and Electronics Engineers, Inc., Piscataway, New Jersey.

For spread spectrum systems, an excellent reference is: *Spread Spectrum Communications Handbook*, by Marvin K. Simon, Jim K. Omura, Robert A. Scholtz, and Barry K. Levitt, McGraw-Hill Inc., New York, 1994.